Intrinsic Thermocouple Analysis Using Multinode Unsteady Surface Element Method

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The intrinsic thermocouple problem has received considerable attention over the last two or three decades. The most common geometry for the intrinsic thermocouple problem is a semi-infinite cylinder (wire) attached perpendicularly to a semi-infinite solid (substrate). In this paper the transient thermal behavior of the interface between the thermocouple wire and the substrate due to step change in the substrate temperature is investigated using the multinode unsteady surface element (MUSE) method. The method uses Duhamel's integral and involves the inversion of a set of Volterra integral equations, one for each surface element. The solution is obtained for a wide range of dimensionless times. The results show excellent agreement between the present solution and the other existing solutions. Very high accuracy is attainable with a relatively small number of surface elements for the complete time domain. This feature makes the method superior to alternative numerical techniques such as finite difference or finite element that involve whole-body discretization.

Nomenclature

a	= radius of the contact area
c	= specific heat
$\bar{F}$	= vector defined by Eq. (17c)
k	= thermal conductivity
M	= time index
N	= number of surface element
q	= heat flux
q_{CL}	= centerline heat flux
$\bar{q}_M$	= heat flux vector at time $t_M = M\Delta t$
r	= radial coordinate
t	= time
t^+	= dimensionless time
T	= temperature
T_c	= contact area temperature
T_c^+	= normalized contact area temperature
$\bar{T}_0$	= initial temperature vector
z	= axial coordinate
α	= thermal diffusivity
β	= variable defined by Eq. (20)
λ	= dummy variable
ρ	= density
ϕ	= temperature rise for unit heat flux (influence function)
$\bar{\bar{\Phi}}_i$	= influence matrix at time $t_i = i\Delta t$

Introduction

INTRINSIC thermocouples are widely used for measuring rapid transient surface temperatures of conducting solids. Such measurements are important in studies of nuclear reactors, laser heating, re-entry vehicle heating, and other applications. An intrinsic thermocouple as defined in Ref. 1 is "one in which the material whose temperature is to be measured (called substrate) forms part of the thermocouple circuit." The most common geometry for the intrinsic thermocouple problem is a semi-infinite cylinder (called wire) attached perpendicularly to a semi-infinite solid (substrate) as shown in Fig. 1.

In this paper, the transient thermal behavior of the interface between the thermocouple wire and the substrate due to a step change in the substrate temperature is analyzed. The multinode unsteady surface element (MUSE) method[2] is employed to obtain the transient solution for the interface heat fluxes and temperatures. The results are compared with those obtained by other investigators for the same problem.

The unsteady surface element method is a new numerical technique,[2-4] which is well suited for the problems involving dissimilar bodies connected over relatively small regions. In this method, only the interface between the two geometries requires discretization as opposed to discretization of the whole domain required in the finite difference (FD) and finite element (FE) methods or discretization of the complete boundary in the boundary integral element (BIE) method.[5-10] This in turn reduces the number of numerical calculations. Further, the method does not require any modifications or special handling of points near the domain boundaries, unlike the above-mentioned alternative methods. The proposed method uses Duhamel's integral and involves the inversion of a set of Volterra integral equations.[2]

Previous Work

The intrinsic thermocouple problem was first studied by Burnett[11] in 1961. He developed an approximate analytical solution for a single wire attached normally to the rear (insulated) surface of a thin slab that was exposed to a constant heat flux on its front face. The effect of the wire was simulated by a uniform disk-shaped sink. Because of the assumption made (for sink strength), the solution provided a conservative estimate for the error in the temperature measurements.[11] Larson[12] and Larson and Nelson[13] improved Burnett's solution. Their solution, like Burnett's, is valid only for the dimensionless time (with respect to the substrate) greater than one.

In 1967, Henning and Parker[14] studied the transient thermal response of an intrinsic thermocouple due to a step

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change in the substrate temperature. They modeled the system as a semi-infinite cylinder attached normal to the surface of a semi-infinite body (called the idealized model). Perfect thermal contact was assumed between the wire and the substrate and all thermal properties were considered temperature independent and constant. It was also assumed that the heat loss from the wire to the surroundings was negligible. To simplify the analysis, they introduced a hemispherical region (shown in Fig. 2) between $r = a$ and $x = 0$ with infinite thermal conductivity and no heat capacity. This assumption resulted in the temperature distributions in both the wire and the substrate being functions of only one space dimension (axial in the wire and radial in the substrate) and time. Utilizing the above assumptions, they derived analytical solutions for the transient temperature distribution in the semi-infinite wire by using the method of separation of variables.

Giedt and Nunn[15] further improved the Henning-Parker solution by applying a modification that provided more accurate values for the early-time response. They stated that the early-time solution of an intrinsic thermocouple can be approximated by using the solution for two semi-infinite bodies (initially at different temperatures) that are suddenly brought together.

In 1971, Bickle[16] and Bickle and Keltner[17] examined the problem by using a combined experimental-numerical method. They developed the "deconvolution" technique to predict the thermocouple response due to an arbitrary substrate temperature variation.

In 1973, Keltner[1] studied the response of a single-wire, intrinsic thermocouple for the case where the substrate undergoes a step temperature change utilizing finite difference and quasicoupling methods. He was first to employ the finite difference procedure to solve the problem for both one- and two-dimensional models.

Because of the deficiencies involved in the finite difference solutions, such as the effort in setting up large grids, large computer expense, and the restricted dimensionless time that could be covered, Keltner developed an alternative method that he called "quasi-coupling." In this procedure, he assumed one-dimensional temperature distributions in both the substrate and the wire and obtained remarkably good agreement with his finite difference solutions. Unlike his finite difference solutions, the quasi-coupled solutions were simple and inexpensive to obtain. The quasi-coupling method is similar in principle to the MUSE method, but is more difficult to implement in practice.

In 1976, Shewen[18] examined the problem by utilizing a two-dimensional finite difference model posed in oblate spheroidal coordinates for the substrate and in cylindrical coordinates for the wire. The two-dimensionality of his solution allowed a more detailed study of the temperature variations along the interface and, consequently, provided more accurate results, particularly at the early times. Keltner and Beck[3] were the first to employ the surface element method to solve the problem. They have considered only one element across the interface and solved the problem analytically by utilizing Laplace transform techniques. Cassange and Bardon[2] considered the same problem using a method similar to that employed by Keltner and Beck.

All of the above-mentioned solution methods (except the two-dimensional finite difference solutions given by Keltner and Shewen) ignored the two-dimensionality of the problem, which is especially important at early times. Even though the two-dimensional finite difference solutions of Keltner and Shewen provided more accurate results, they are not entirely satisfactory. They have some difficulties due to the necessity of setting up extremely fine grids near the interface and many large grids further from the interface. In turn, this increases the number of numerical calculations and the computer time.

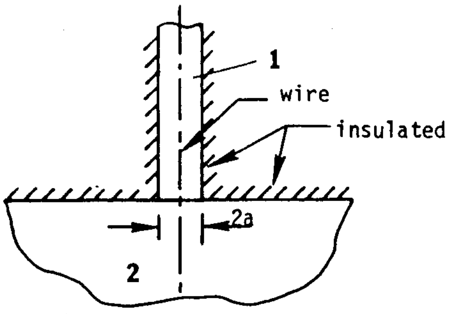

Fig. 1 Intrinsic thermocouple.

Statement of the Problem

The idealized geometry for the intrinsic thermocouple problem is shown in Fig. 1. The semi-infinite cylinder is referred to as region 1 and the semi-infinite substrate as region 2. At zero time the entire substrate undergoes a step change of temperature to T_{02}, while the wire is at its initial temperature of T_{01}. The bodies are assumed to be in perfect contact and to have temperature-independent thermal prop-

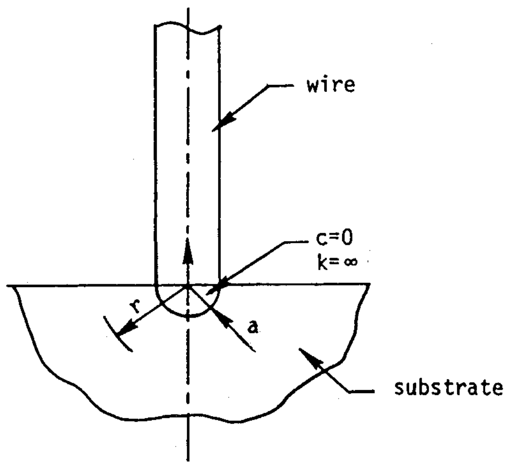

Fig. 2 Henning-Parker idealized geometry for an intrinsic thermocouple problem.

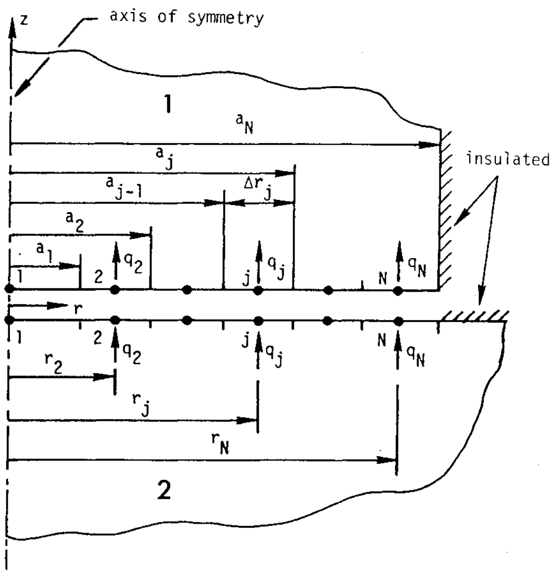

Fig. 3 Discretization of the interface between semi-infinite thermocouple wire and semi-infinite substrate.

erties. The describing differential equations are

$$\frac{\partial^2 T_1}{\partial r^2}+\frac{1}{r}\frac{\partial T_1}{\partial r}+\frac{\partial^2 T_1}{\partial z^2}=\frac{1}{\alpha_1}\frac{\partial T_1}{\partial t}$$

$$\frac{\partial^2 T_2}{\partial r^2}+\frac{1}{r}\frac{\partial T_2}{\partial r}+\frac{\partial^2 T_2}{\partial z^2}=\frac{1}{\alpha_2}\frac{\partial T_2}{\partial t} \quad (1)$$

$$T_1=T_{01} \text{ for } t=0, \quad 0\le r\le a, \quad z\ge 0 \quad (2a)$$

$$T_2=T_{02} \text{ for } t=0, \quad r\ge 0, \quad z\le 0 \quad (2b)$$

$$T_1=T_{01} \text{ for } t>0, \quad 0\le r\le a \text{ as } z\to\infty \quad (3a)$$

$$T_2=T_{02} \text{ for } t>0 \text{ as } r\to\infty \text{ and } z\to-\infty \quad (3b)$$

$$\frac{\partial T_1}{\partial r}=\frac{\partial T_2}{\partial r}=0 \text{ at } r=0 \text{ for } t>0 \quad (4)$$

$$T_1=T_2, \quad k_1\frac{\partial T_1}{\partial z}=k_2\frac{\partial T_2}{\partial z} \text{ for } t>0, \quad 0\le r\le a, \quad z=0 \quad (5)$$

$$\frac{\partial T_1}{\partial r}=0 \text{ for } t>0, \quad r=a, \quad z\ge 0 \quad (6a)$$

$$\frac{\partial T_2}{\partial z}=0 \text{ for } t>0, \quad r>a, \quad z=0 \quad (6b)$$

where T_1 and T_2 denote the temperature distributions, k_1 and k_2 the thermal conductivities, α_1 and α_2 the thermal diffusivities, r and z the space coordinates, and t the time.

Surface Element Formulation

Due to the axisymmetric nature of the above problem, the contact area between the wire and the half-space is divided into N annular surface elements with each of these elements having inner and outer radii denoted by a_{j-1} and a_j, respectively ($j=1,\ldots,N$ and $a_0=0$). In Fig. 3 notice that only the parts of the boundary with nonzero values of heat flux need to be discretized. In general, the heat flux and the temperature vary across the interface, but they are assumed to be uniform over each surface element (linear and quadratic variations are also possible) and are specified at the points

$$r_1=0; \quad r_j=(a_j-a_{j-1})/2, \quad j=2, N \quad (7)$$

The heat flux $q_j(t)$ (associated with element j) leaving body 2 in Fig. 3 is the same heat flux that enters body 1 over the region $r=a_{j-1}$ to $r=a_j$, that is,

$$-k_1\frac{\partial T_1}{\partial z}=-k_2\frac{\partial T_2}{\partial z} \text{ for } t>0, \; a_{j-1}\le r\le a_j, \; z=0 \quad (8)$$

Using Duhamel's integral, the temperature at element k in body 1 and time t can be given by,

$$T_{k1}(t)=T_{01}+\sum_{j=1}^{N}\int_0^t q_j(\lambda)\frac{\partial\phi_{kj}^{(1)}(t-\lambda)}{\partial t}d\lambda \quad (9)$$

where $\phi_{kj}^{(1)}(t)$ is the temperature rise at element k and time t due to unit step heat flux over element j of surface 1. It is the basic building block needed for body 1 and is called the influence function of body 1.

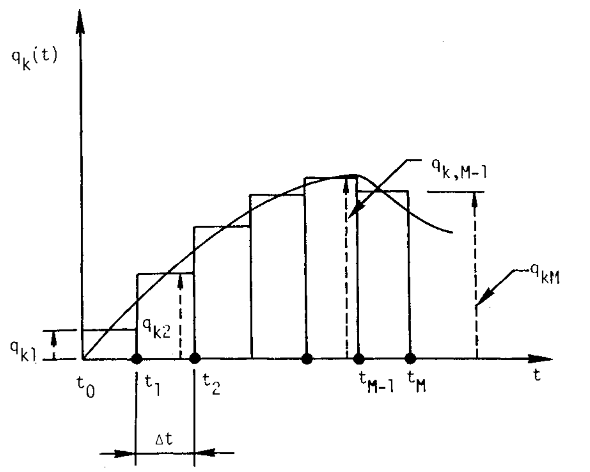

Fig. 4 Geometry illustrating uniform heat flux assumption over each time interval.

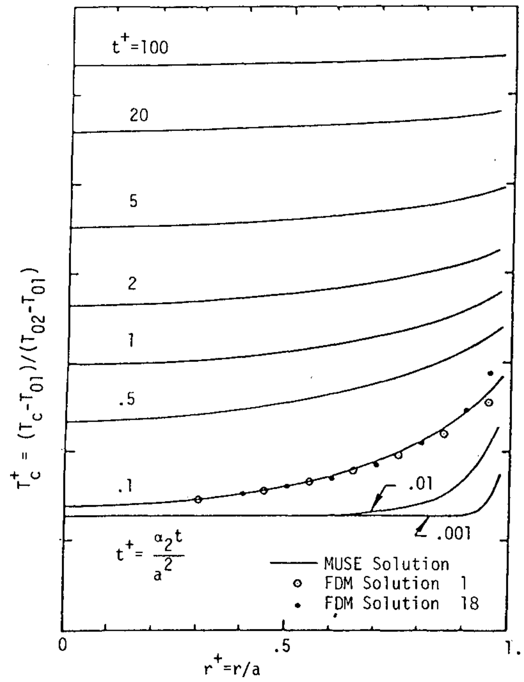

Fig. 5 Normalized interface temperature distribution.

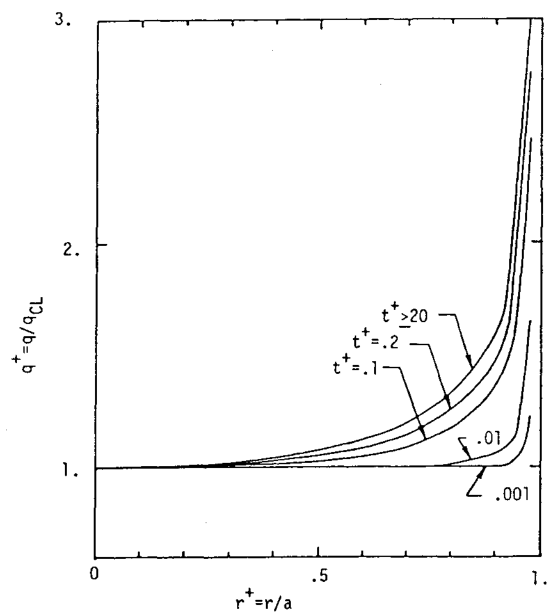

Fig. 6 Normalized heat flux distribution across the interface at various values of time.

Similar to Eq. (9), an integral equation can be given for the kth surface element of body 2,

$$T_{k2}(t) = T_{02} - \sum_{j=1}^{N} \int_0^t q_j(\lambda) \frac{\partial \phi_{kj}^{(2)}(t-\lambda)}{\partial t} \mathrm{d}\lambda \tag{10}$$

where $\phi_{kj}^{(2)}(t)$ is the influence function for body 2. The minus sign before the summation in Eq. (10) is used because the heat flux is pointing outward from body 2. For the case where the bodies are in perfect contact, one can write

$$T_{k1}(t) = T_{k2}(t) \quad \text{for } k = 1,2,\ldots,N \tag{11}$$

By introducing Eqs. (9) and (10) into Eq. (11), a set of integral equations for $k = 1,2,\ldots,N$ can be found,

$$T_{02} - T_{01} = \sum_{j=1}^{N} \int_0^t q_j(\lambda) \frac{\partial \phi_{kj}(t-\lambda)}{\partial t} \mathrm{d}\lambda \tag{12}$$

where

$$\phi_{kj}(t) = \phi_{kj}^{(1)}(t) + \phi_{kj}^{(2)}(t) \tag{13}$$

Equation (12) represents a set of Volterra integral equations of the first kind that can be solved simultaneously for N unknown heat flux histories $q_1(t)$, $q_2(t)$,...,$q_N(t)$.

Solution of the Simultaneous Integral Equations

The Volterra equation (12) can be approximated by a system of linear algebraic equations by replacing the integrals with suitable quadrature formulas. As the first step, the time region between 0 and t is divided into M equally small time intervals Δt so that t_M represents the value of t at the end point of the Mth interval ($t_M = M\Delta t$). Further, in the simplest form of approximation, the heat flux histories $q_j(t)$ are assumed to have constant values in each time interval as shown in Fig. 4. Then Eq. (12) can be written as

$$T_0 = \sum_{j=1}^{N} \sum_{j=1}^{N} q_{ji}(\phi_{kj,M+1-i} - \phi_{kj,M-i}) \quad \text{for } k = 1,2,\ldots,N \tag{14}$$

where

$$T_0 = T_{02} - T_{01} \tag{15a}$$

$$q_{ji} = q_j(t_i) \tag{15b}$$

$$\phi_{kji} \equiv \phi_{kj}(t_i) \tag{15c}$$

In the form given by Eq. (14), the heat fluxes q_{jM} (for $j = 1,\ldots,N$) can be determined at different time intervals one after another by marching forward in time for $M = 1,2,3\ldots$; that is, for each time step, Eq. (14) represents a system of N equations with N unknowns, q_{1M}, q_{2M},...,q_{NM}.

Expressing Eq. (14) in matrix form with the unknowns on the left and the knowns on the right gives

$$\bar{\bar{\Phi}}_1 \bar{q}_M = \bar{T}_0 + \sum_{i=1}^{M-1} \bar{\bar{\Phi}}_{M-i} \bar{q}_i - \sum_{i=1}^{M-1} \bar{\bar{\Phi}}_{M+1-i} \bar{q}_i \tag{16}$$

where $\bar{T}_0$ is the initial temperature vector and $\bar{\bar{\Phi}}_i$ and $\bar{q}_i$ the influence matrix and the heat flux vector at time t_i, respectively. The elements of the influence matrix $\bar{\bar{\Phi}}_i$ are given by Eq. (15c).

At each time step, the solution of Eq. (16) yields the flux distribution q_j ($j = 1,\ldots,N$) that will produce the elemental temperatures over the contact area. Solving Eq. (16) for the unknowns $\bar{q}_M$ ($M = 1,2,\ldots$) yields

$$\bar{q}_1 = \bar{\bar{\Phi}}_1^{-1} \bar{T}_0 \tag{17a}$$

$$\bar{q}_M = M\bar{q}_1 - \bar{\bar{\Phi}}_1^{-1} \bar{F}_M \quad \text{for } M = 2,3,\ldots \tag{17b}$$

where

$$\bar{F}_M = \sum_{i=1}^{M-1} \bar{\bar{\Phi}}_{M+1-i} \bar{q}_i \tag{17c}$$

Notice that the column matrix $\bar{F}_M$ is the only term that should be evaluated at each time step. The influence function for the half-space and the wire can be found from the known available solutions of a semi-infinite body heated by a constant disk heat source[19] and a semi-infinite insulated cylinder heated by a constant heat flux over a disk area centered at the end,[20] respectively.

Utilizing Eqs. (17) the intrinsic thermocouple problem is solved using the thermal properties of a chromel substrate and an alumel wire. The chromel and alumel combination has also been investigated in Refs. 1, 3, and 18. The thermal conductivities k are 19.21 and 29.76 W/m·K and the thermal diffusivities α are 0.492×10^{-5} and 0.663×10^{-5} m²/s for

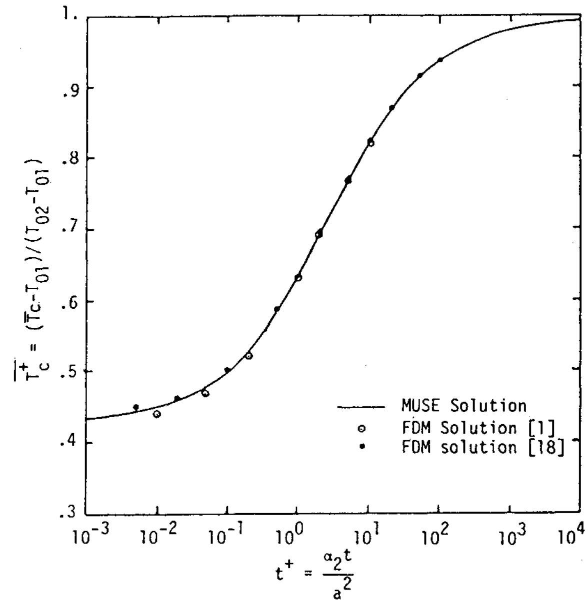

Fig. 7 Normalized area-averaged interface temperature histories.

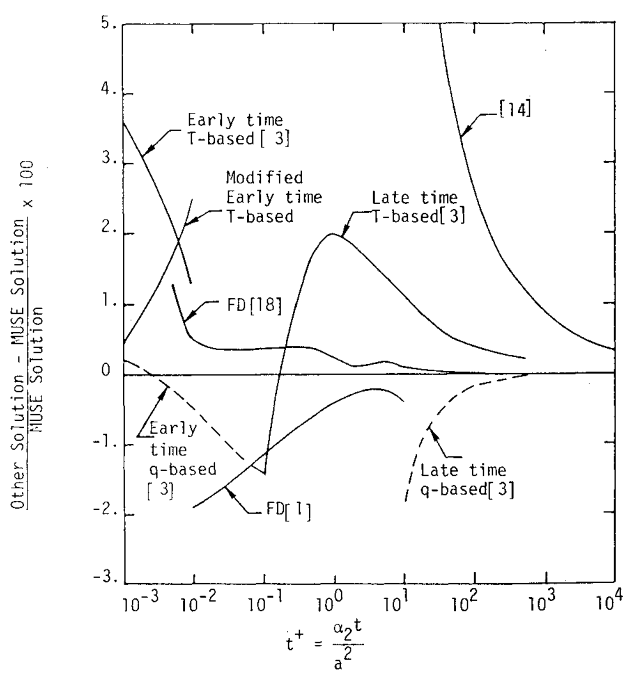

Fig. 8 Percent of deviation (with respect to MUSE solution) in interface temperature $\bar{T}_c^+$ as a function of time for intrinsic thermocouple problem.

Table 1 Normalized area-averaged interface temperature histories for chromel substrate and alumel wire

	FD solutions			T-based, USE[a] solutions[3]		q-based, USE[a] solutions		
t^+	Ref. 1	Ref. 18	Ref. 14	Early time	Late time	Early time	Late time	MUSE
0.001			0.6084	0.4489		0.4342		0.4335
0.002		0.4421	0.6118	0.4500		0.4366		0.4364
0.005		0.4480	0.6185	0.4521		0.4413		0.4422
0.01	0.4402	0.4510	0.6257	0.4545		0.4467		0.4488
0.02	0.4505	0.4599	0.6356			0.4546		0.4581
0.05	0.4700	0.4782	0.6540			0.4709		0.4765
0.1	0.4916	0.4916	0.6731		0.4907	0.4904		0.4973
0.2	0.5215	0.5283	0.6972		0.5280			0.5263
0.5	0.5770	0.5826	0.7373		0.5910			0.5805
1.0	0.6302	0.6338	0.7729		0.6452			0.6328
2.0	0.6896	0.6921	0.8109		0.7042			0.6915
5.0	0.7688	0.7714	0.8602		0.7810			0.7700
10.0	0.8202	0.8246	0.8933		0.8327		0.8091	0.8236
20.0		0.8694	0.9207		0.8757		0.8614	0.8687
50.0		0.9139	0.9482		0.9186		0.9108	0.9137
100.0		0.9382	0.9629		0.9417		0.9365	0.9381
200.0			0.9735		0.9585		0.9550	0.9559
500.0			0.9832		0.9737		0.9715	0.9719

[a]Unsteady surface element.

chromel and alumel, respectively. (The results presented are actually valid for any material with the ratios of $k_2/k_1 = 0.645$ and $\alpha_2/\alpha_1 = 0.742$.)

The contact area between the wire and the substrate is divided into 10 annular nonequally spaced elements, with the smaller elements being closer to the corner region. The problem is solved for the elemental surface temperatures and heat fluxes for various values of dimensionless time, ranging between $t^+ = 0.001$ and 10^4. The dimensionless time is based on the thermal diffusivity of the substrate (body 2),

$$t^+ = \alpha_2 t/a^2 \tag{18}$$

The results are compared with those obtained from Henning and Parker's solution,[14] Keltner's model III finite difference solution,[1] Shewen's finite difference solution,[18] and Keltner and Beck's single node surface element solution[3] on the basis of the area-averaged interface temperature.[1]

Results and Discussion

The results of the surface element solution are presented in terms of the temperature and heat flux distributions across the interface and the area-averaged interface temperature and heat flux as functions of time. Figure 5 shows the normalized spatial variation of the interface temperature at different dimensionless times. Normalization is obtained by subtracting T_{01} from elemental values and dividing the results by $(T_{02} - T_{01})$. It can be seen from Fig. 5 that the temperature gradient in the region near the corner of the interface is very large at early time and approaches zero as t^+ goes to infinity. This demonstrates the two-dimensionality of the problem and also indicates that more accurate studies are required in this region, especially at the early times.

In his model III finite difference solution, Keltner used more than 400 nodes in the semi-infinite body (and about 150 in the wire) to represent the corner region effectively. He found that even his fine-grid structure introduces some errors. Employing oblate spheroidal coordinates, Shewen[19] obtained approximately the same results with a smaller number of nodes (120 as reported in Ref. 18) and consequently less computational effort than that used by Keltner. Their results are also shown in Fig. 5 for time $t^+ = 0.1$. It can be seen that the agreement between these finite difference solutions and the present surface element solution is very good. Notice that the number of nodes used in the MUSE solution is only 10, 1/12 as many as those used by Shewen and about 1/55 as many as used by Keltner.

When the substrate initially undergoes a step change of temperature, there is an instantaneous change to a common interface temperature that depends upon the thermal properties of the substrate and the wire.‡ The normalized value of this initial interface temperature§ is[3]

$$T_c^+ = (T_c - T_{01})/(T_{02} - T_{01}) = (1+\beta)^{-1} \tag{19}$$

where

$$\beta = (k_1 \rho_1 c_1 / k_2 \rho_2 c_2)^{1/2} \tag{20}$$

For a chromel substrate and an alumel wire, $T_c^+ = 0.4285$. After the initial moment, the effect of the edge starts to penetrate toward the center of the contact area. At the early time $t^+ = 0.001$, the interface temperature is almost at its initial value (0.4285) for $r^+ \leq 0.83$. The only part of the contact area disturbed by the edge effect is between $r^+ = 0.83$ and 1. The edge effect penetrates further toward the center as t^+ becomes larger, reaching $r^+ = 0.45$ at $t^+ = 0.01$ and covering the entire area for $t^+ > 0.02$. The difference between the centerline and the outermost element temperatures is about 10.7% at $t^+ = 0.001$ and decreases as t^+ becomes larger. For $t^+ > 20$, the interface temperature distribution becomes almost uniform, which indicates that the one-dimensional approximate solution given by Henning and Parker[14] and the T-based solution of Keltner and Beck[3] are appropriate for the late times.

Figure 6 shows the normalized heat flux distribution across the interface $q^+(t)$ at several times. It can be seen that the region of uniform heat flux shrinks as t^+ increases. After the dimensionless time of 20, the q^+ distribution remains constant and a universal curve is obtained.

Figure 7 shows the normalized area-averaged interface temperature vs the dimensionless time. Results for the model III finite difference solution of Ref. 1 and the finite dif-

‡Notice that at the initial moment there is no spatial variation in the interface temperature.

§This temperature corresponds to the surface temperature of two semi-infinite bodies initially at different temperatures suddenly brought into perfect contact.

ference solution of Ref. 18 as well as the MUSE solution are presented. Table 1 provides the results of the above-mentioned three solutions along with the results from the approximate analytical solution of Ref. 14 and the *T*-based and the *q*-based solutions of Ref. 3. The first column in this table is the dimensionless time extending from $t^+ = 0.001$ to 500. The results of the finite difference solutions of Refs. 1 and 18 are given in the second and the third columns, respectively. The fourth column is evaluated from Eq. (22) given by Henning and Parker,[14] which is good only for the late time $t^+ > 20$. The early- and the late-time results of the *T*-based solution and the *q*-based solution of Keltner and Beck are displayed in the next four columns. The last column represents the present MUSE solution. As can be seen from Fig. 7 and Table 1, there is very good agreement between the finite difference solutions and the present solution for the time range covered. However, as mentioned earlier, both finite difference solutions have difficulties regarding the computational effort and cost, particularly for the early times $t^+ < 0.01$ and late times $t^+ > 10$.

The *T*- and *q*-based single-node surface element solutions given by Keltner and Beck[3] are convenient in that the mathematics is not difficult and the expressions are simple to evaluate. Each solution provides two expressions: one for early times and the other for late times. The *q*-based solution is more appropriate for the early times. It approaches the exact solution (0.4285) as $t^+ \to 0$ and closely matches the MUSE solution up to dimensionless time $t^+ = 0.1$. It also provides relatively good results for the late times $t^+ > 10$. The *T*-based solution does not approach the exact solution as $t^+ \to 0$ and, consequently, it is less accurate than the *q*-based solution for the early times. Because of the constant interface temperature assumption, however, it yields very good results for the late times of $t^+ > 0.1$. It should be noted that neither the *T*-based solution nor the *q*-based solution is solely suitable for the complete time domain. However, a combination of the early-time *q*-based solution [Eq. (41a) of Ref. 3] and the late-time *T*-based solution [Eq. (33a) of Ref. 3] provides very good results over the entire time range. These two solutions match very closely at dimensionless time $t^+ = 0.1$. Figure 8 shows the percent of deviation between the above-mentioned solutions[1,3,14,18] and the present solution. It can be seen that the finite difference solution given by Shewen[18] is in the best agreement for $t^+ > 0.002$ within approximately 1%. The model III finite difference solution of Keltner has a deviation of about 2% at $t^+ = 0.01$, which decreases to less than 1% at late times. The solution given by Henning and Parker[14] is good for late times, but is in poor agreement for early-to-mid times less than 20. It shows a 6% difference at $t^+ = 20$, which increases to 40.3% at $t^+ = 0.001$. All of the approximate solutions presented by Keltner and Beck lie within 2% of the present solution, which shows a deviation of about 4.2% at time zero. This solution can be modified by letting the factor of 1.90484 [given in Eq. (34) of Ref. 13] be replaced by $\pi^{1/2}$. The modified solution provides better results for the early times ($t^+ < 0.005$) and approaches the exact solution as $t^+ \to 0$. See Fig. 8.

A study of the above results shows that the multinode surface element solution provides an accurate representation of the idealized intrinsic thermocouple. It is superior to other available solutions in terms of accuracy and ability to treat the complete time domain. The method is most suitable for calculating the interface temperature and heat flux, particularly at early times when the two-dimensionality of the problem is significant.

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